

Name: \_\_\_\_\_  
Student ID: \_\_\_\_\_

## Test 2

This test is graded out of 48 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** (5 marks) Find the derivative of the following function and simplify:

$$f(x) = \frac{(3x^2 + 2)^5}{(5x + 1)^6}$$

**Question 2.** (5 marks) Find the derivative of the following function:

$$g(t) = t^3 + \frac{1}{\sqrt{t}} + 2 \sin t + 3 \csc t + 4 \arctan t$$

**Question 3.** (4 marks) Find the derivative of the following function:

$$h(z) = \sqrt{\operatorname{arcsec}(z^2 + 1)}$$

**Question 4.** (4 marks) Find the derivative of the following function:

$$f(x) = (3x^3 + x)^{3/2} \sqrt{2x - 1}$$

**Question 5.** The yearly demand for the Super Cool Fixed Gear bicycle is

$$p = 1000 - 0.04x \quad (0 \leq x \leq 20000)$$

where  $p$  denotes the wholesale unit price in dollars and  $x$  denotes the quantity demanded. The weekly total cost function associated with manufacturing the Super Cool Fixed Gear is given by

$$C(x) = 0.000003x^3 - 0.02x^2 + 300x + 70000$$

where  $C(x)$  denotes the total cost incurred in producing  $x$  Super Cool Fixed Gear.

- (2 marks) Find the revenue function  $R$  and the profit function  $P$ .
- (3 marks) Find the marginal cost function  $C'$ , the marginal revenue function  $R'$  and the marginal profit function  $P'$ .
- (2 marks) Compute  $P'(2000)$  and interpret your results.

**Question 6.** (6 marks) Find the third derivative of the following function. Simplify your final answer.

$$f(x) = \frac{x^2 + 1}{(2x + 1)^2}$$

**Question 7.** Using the demand function:

$$p = 1000 - 0.04x$$

- a. (3 marks) Calculate the elasticity of demand function  $E(p)$ .
- b. (2 marks) Calculate the elasticity of demand when the price is set at \$750. Interpret the result.
- c. (1 mark) At what price is the demand unitary?

**Question 8.** Given:

$$6 + \sqrt{2} - 2xy = \sqrt{y+1}$$

- a. (5 marks) Find  $\frac{dy}{dx}$ .
- b. (2 marks) Find the equation of the tangent to the graph of the given relation at  $(1, 3)$

**Question 9.** (5 marks) Find the equation of the tangent to the function  $f(x) = \tan 2x$  at  $x = \frac{\pi}{8}$ .

**Bonus Question.** (5 marks)

Find the derivative of the following function but do *not* simplify.

$$f(x) = \cot \left( \sqrt{\frac{\arcsin(\tan x + x^2) + x}{\operatorname{arccot}(\sec x + e) - x}} + \pi \right)$$