Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S2: Winter 2012

Name: ____________________

Student ID: ____________________

Test 1

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formulae:

$\sum_{i=1}^{n} c = cn$ where c is a constant $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(5 marks)* Find the average of the function $f(x) = -6x^2 + 2x + 1$ on $[-1, 2]$ using the definition of the definite integral.

Question 2. *(5 marks)* Evaluate the definite integral:

$$\int_{-\sqrt{3}}^{1} |\arctan x| \, dx$$

Question 3. *(5 marks)* Evaluate the definite integral:

$$\int_{-1}^{1} \frac{x^2 - x}{2x^3 - 3x^2 - 101} + \frac{x^2 \sin x}{1 + x^6} \, dx$$

Question 4. *(5 marks)* Evaluate the indefinite integral:

$$\int \frac{e^x \arcsin(e^x - 1)}{\sqrt{2e^x - e^{2x}}}\, dx$$

Question 5. *(5 marks)* Evaluate the expression and simplify:

$$\frac{d}{dx}\left[\int_{\ln(2x)}^{\ln(x^2)} u^{e^u}\, du\right]$$

Question 6. *(5 marks)* Evaluate the indefinite integral:

$$\int x\csc^2 2x\,dx$$

Question 7. *(5 marks)* Estimate the area under the graph of $f(x) = \arccos x$ from $x = -1$ to $x = 1$ using two rectangles and using the left endpoints. Sketch the curve and the approximating rectangles. Is the estimate an overestimate or underestimate? Justify.

Question 8. *(5 marks)* Prove: If $f(x)$ is an even integrable function on $[-a,a]$ then

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

Question 9.

a. *(1 mark)* Show that

$$\int f(x)\,dx = xf(x) - \int xf'(x)\,dx$$

b. *(4 marks)* If f and g are inverse functions *(that is, $f(g(x)) = x$ and $g(f(x)) = x$)*and f' is continuous, prove that

$$\int_{a}^{b} f(x)\,dx = bf(b) - af(a) - \int_{f(a)}^{f(b)} g(y)\,dy$$

(hint: you may use part a.)

Bonus Question. *(3 marks)*
Evaluate:

$$\lim_{h\to 0}\frac{\lim\limits_{n\to\infty}\left[\frac{x+h-\pi}{n}\left[\left(\pi+\frac{x+h-\pi}{n}\right)^e+\left(\pi+2\frac{x+h-\pi}{n}\right)^e\ldots+\left(\pi+n\frac{x+h-\pi}{n}\right)^e\right]\right]-\lim\limits_{n\to\infty}\left[\frac{x-\pi}{n}\left[\left(\pi+\frac{x-\pi}{n}\right)^e+\left(\pi+2\frac{x-\pi}{n}\right)^e+\ldots+\left(\pi+n\frac{x-\pi}{n}\right)^e\right]\right]}{h}$$