# 4.7 Antiderivatives

Definition: A function $F$ is called an **antiderivative** of $f$ on an interval $I$ if $F'(x) = f(x)$ for all $x$ in $I$

Notice that $G(x) = \frac{1}{3}x^3 + x^2 + 2$, $H(x) = \frac{1}{3}x^3 + x^2$ AND $F(x) = \frac{1}{3}x^3 + x^2 - 5$ are all antiderivatives of $f(x)$ since $G'(x) = H'(x) = F'(x) = f(x)$
These functions only differ by a constant. In fact if $F(x)$ and $G(x)$ are antiderivatives of $f(x)$ then

$F(x) - G(x) = C$ SO $G(x) = F(x) + C$

So if $F(x)$ is any antiderivative of $f(x)$ then all antiderivatives have the form $F(x) + C$

The process of finding all antiderivatives is called **antidifferentiation** or **indefinite integration.**

Notation:

Basic Rules of Integration

1) $\int k \, dx$

2) $\int c f(x) \, dx$

3) $\int f(x) \pm g(x) \, dx$

4) $\int x^n \, dx$

5) $\int \frac{1}{x} \, dx$

Examples:

1) $\int (4x^5 - 3x^2 + 5x + 6) \, dx$

2) $\int \frac{x^2 + 2x^{-1/2} - 5}{\sqrt{x}} \, dx$

Since we know the derivatives of other functions we can use these to make more integration rules:

6) $\int e^x \, dx$

7) $\int \cos x \, dx$

8) $\int \sin x \, dx$

9) $\int \sec^2 x \, dx$

10) $\int \sec x \tan x \, dx$

11) $\int \csc^2 x \, dx$

12) $\int \csc x \cot x \, dx$

13) $\int \frac{1}{\sqrt{1-x^2}} \, dx$

14) $\int \frac{1}{1+x^2} \, dx$

15) $\int \frac{1}{x\sqrt{x^2-1}} \, dx$

WHY DON'T WE NEED $\int \frac{-1}{\sqrt{1-x^2}} \, dx = \cos^{-1} x + C$ ?

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Examples:

1) FIND ALL FUNCTIONS $g(x)$ SUCH THAT

$$g'(x) = 3\sin x + (\sqrt{x} - 2x)(4x^2 - 7) + 7e^x - \frac{4x}{x^3 + x} + 3\sec x \tan x$$

2) FIND $f$ IF $f'(x) = e^x + 20(1+x^2)^{-1}$ AND $f(0) = -2$

Ex: Find the most general antiderivatives of the following functions

1) $f(x) = \sqrt[3]{x^2} + x\sqrt{x}$

2) $f(t) = \dfrac{3t^4 - t^3 + 6t^2}{t^4}$

3) $r(\theta) = \sec\theta\tan\theta - 2e^\theta$

Ex: Find f.

1) $f'''(t) = e^t + t^{-4}$

2) $f''(t) = 2e^t + 3\sin t, \quad f(0) = 0 \quad , \; f(\pi) = 0$

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