Other Convergence Tests

An **alternating series** is a series where the terms alternate from positive to negative.

$$\sum_{n=k}^{\infty}(-1)^n b_n \qquad \text{or} \qquad \sum_{n=k}^{\infty}(-1)^{n+1} b_n$$

where $b_n \geq 0$.

For example:

Theorem: If the alternating series

$$\sum_{n=1}^{\infty}(-1)^{n+1} b_n = b_1 - b_2 + b_3 - b_4 + \ldots \qquad (b_n > 0)$$

satisfies i) $b_{n+1} \leq b_n$ for all n

ii) $\lim_{n\to\infty} b_n = 0$

then the series is convergent.

Proof:

Example: Determine if the following series are convergent or divergent:

1) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

2) $\sum_{n=8}^{\infty} (-1)^n \frac{n}{n^2+1}$

3) $\sum_{n=20}^{\infty} (-1)^{n+1} \left[\ln(en) - \ln(en+1)\right]$

A series $\sum a_n$ is called **absolutely convergent** if the series of absolute values $\sum |a_n|$ is convergent.

Examples: 1) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$

2) $\displaystyle\sum_{n=1}^{\infty} n^{-1/2}$

A series $\sum a_n$ is called **conditionally convergent** if it is convergent but not absolutely convergent.

Example: 1) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

Theorem: If a series $\sum a_n$ is absolutely convergent then it is convergent.

Proof: