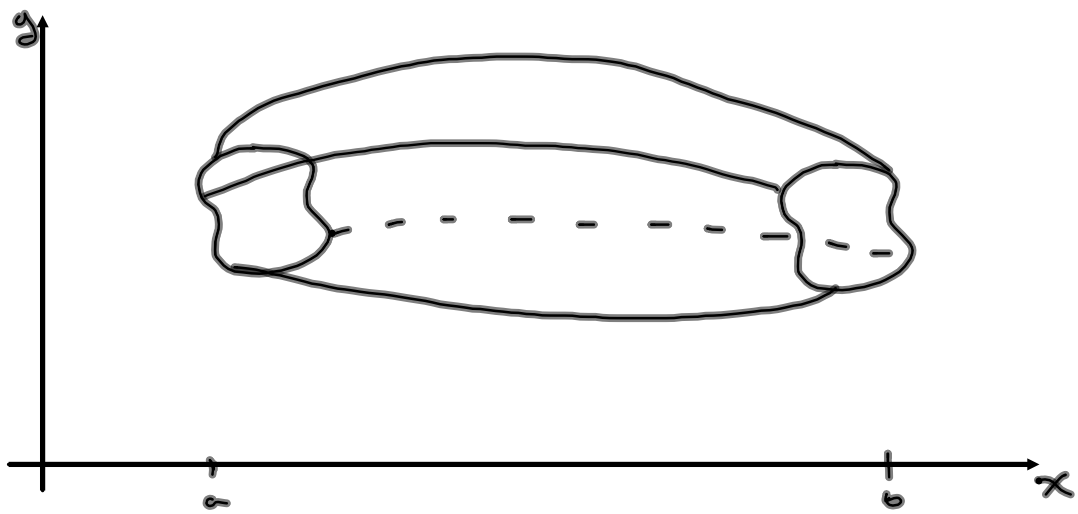
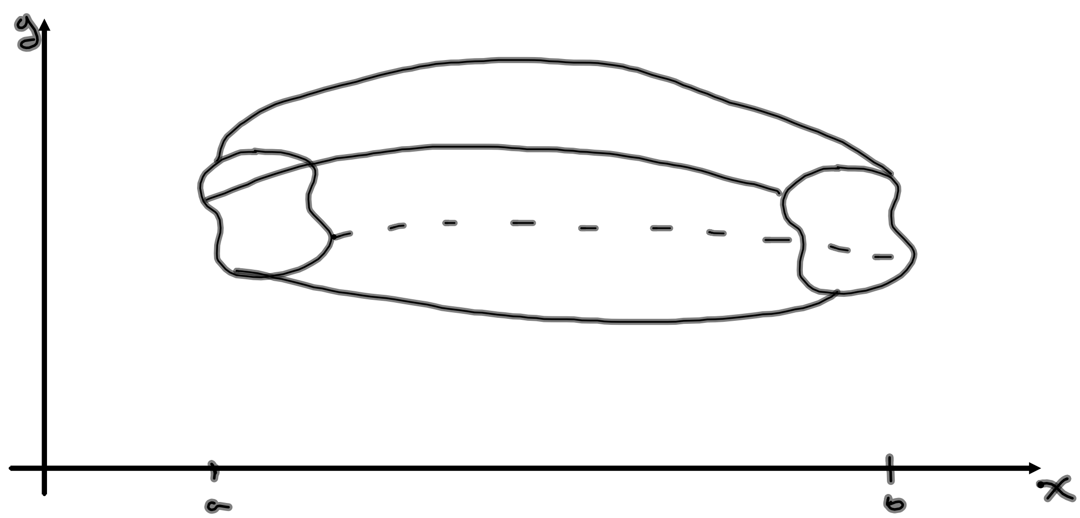
7.2 Volumes

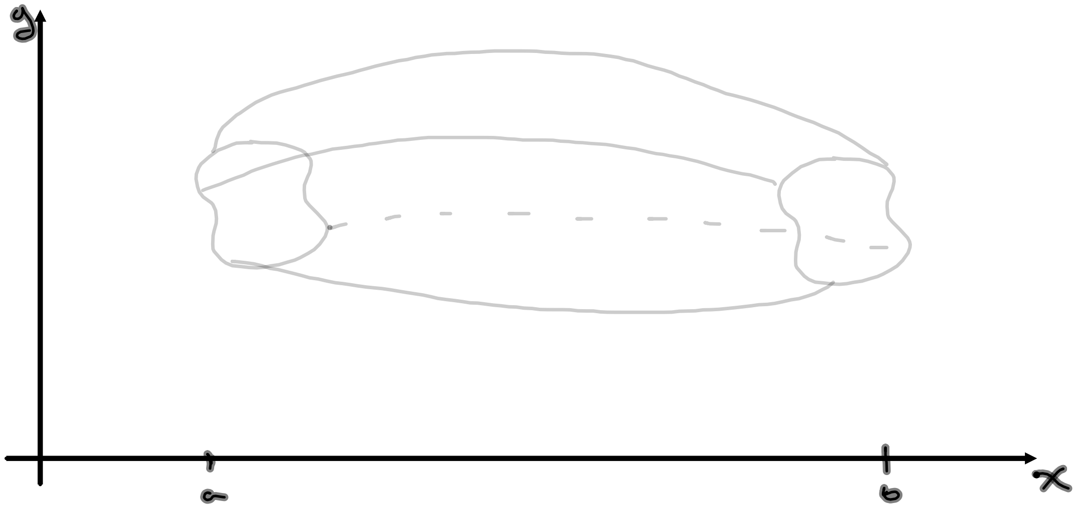
Suppose we want to find the volume of the following solid



If we intersect the solid, S, with a plane we obtain a **cross-section**. Let's call the area of this cross-section $A(x)$.

If we consider a partition of the interval [a,b] into n subintervals with partition points $x_1, x_2, x_3, \ldots, x_n$ we can approximate the volume using n "slabs" of width $\triangle x = x_i - x_{i-1}$.





Each slab has volume $V_i = A(x_i^*)\Delta x$

Adding the volumes together we get an approximation

The approximation gets better as the slices become thinner (that is as $\max\Delta x \rightarrow 0$) so we define the following.

<u>Definition of Volume</u>

Let S be a solid that lies between $x = a$ and $x = b$. If the cross sectional area of S in the plane through x and perpendicular to the x-axis is $A(x)$, where A is an integrable function, then the volume of S is

Examples: 1) Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = \sqrt{\sin x}$ and the x-axis on $0 \leq x \leq \pi$ about the x-axis.

2) Find the volume of the solid obtained by rotating the region under the curve $y = \sqrt{x}$ from 0 to 1.

3) Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ about the y-axis.

4) Find the volume of the solid formed by revolving the region bound by $f(x) = 2 - x^2$ and $g(x) = 1$ about the line $y = 1$.

5) Find the volume of the solid formed by rotating the region bounded by the graphs $y = \sqrt{x}$ and $y = x^2$ about the x-axis.

6) Find the volume of the solid obtained by rotating the region enclosed by the curves $y = x$ and $y = x^2$ about the line $y = 2$.