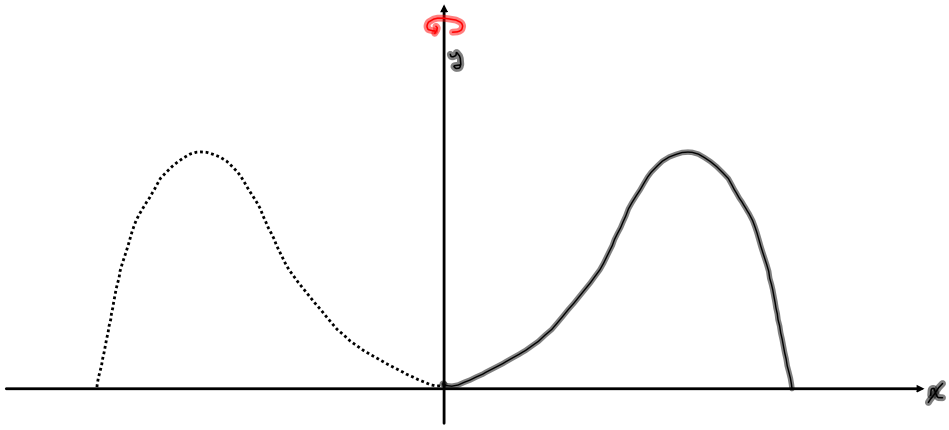
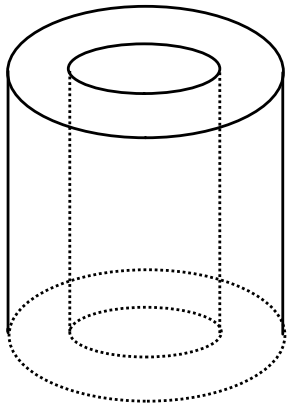
7.3 Volumes by Cylindrical Shells

Suppose we wanted to find the volume of the solid formed by rotating the region bounded by $y = 2x^2 - x^3$ and $y = 0$ about the y-axis.

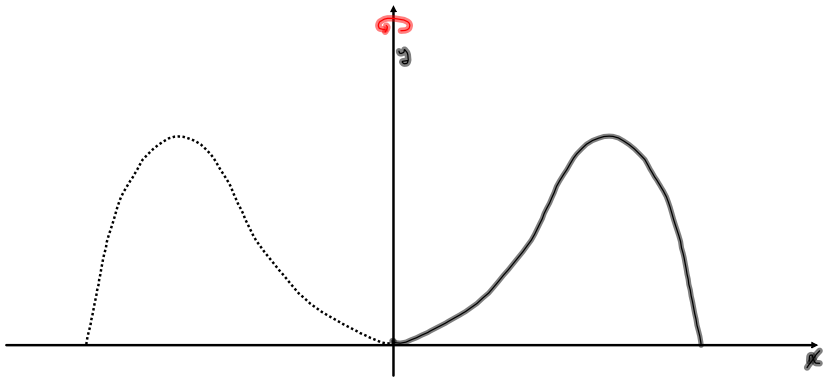


Let's develop another method. Again, we start with an approximation but this time we will approximate using cylindrical shells.

Let's calculate the volume of a cylindrical shell.



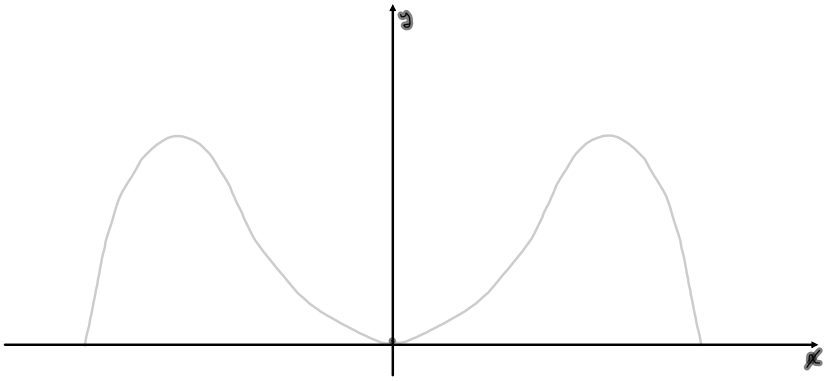
Let's use this to approximate the following volume



Partition [a,b] into n subintervals with partition points $x_0, x_1, x_2, \ldots, x_n$

The volume of a shell can be written as

And so our approximation looks like



And can be written as

Again, our approximation gets better as $\max \triangle x \longrightarrow 0$

It turns out that

Examples: 1) Find the volume of the solid obtained by rotating the region bounded by $y = 2x^2 - x^3$ and $y = 0$ about the y-axis.

2) Find the volume of the solid generated by rotating the region bounded by $y = \sqrt{x}, x = 1, x = 4$ and the x-axis is revolved about the y-axis.

3) Find the volume of the solid generated by rotating the region in the first quadrant enclosed between $y = x$ and $y = x^2$.

4) Find the volume of the solid generated when the region R under $y = x^2$ over the interval [0,2] is revolved about the x-axis.

5) Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.