Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S09: Winter 2013

Name: _____
Student ID: _____

Test 1

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formulae:

$\sum_{i=1}^{n} c = cn$ where c is a constant $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(5 marks)* Evaluate the definite integral of $f(x) = -6x^2 + 4x + 3$ on $[-2, 1]$ using the definition of the definite integral.

Question 2. *(5 marks)* Evaluate the integral by interpreting it in terms of areas.

$$\int_{-3}^{0} 1+\sqrt{9-x^2}\,dx$$

Question 3. *(5 marks)* If

$$\int_{0}^{\pi} f(x)\,dx = \sqrt{2}$$

and $f(x) = f(-x)$ for all $x \in \mathbb{R}$ then evaluate

$$\int_{-\pi}^{\pi} 2f(x) + \frac{x^2 \sin x}{1+x^6}\,dx$$

Question 4. *(5 marks)* Evaluate the indefinite integral:

$$\int \frac{\csc 3x \cot 3x}{1+\csc^2 3x}\, dx$$

Question 5. Given

$$h(x) = \int_{2e^x}^{e^{2x}} u^u \, du$$

a. *(2 marks)* Rewrite $h(x)$ as the sum of two integrals with a constant as the lower bound.

b. *(1 mark)* Rewrite the two integrals of part a. as composite functions with an integral as the outer function.

c. *(2 marks)* Using part b. and the 2nd FTC determine $h'(x)$.

Question 6. Given

$$f(x) = \frac{1}{x}, \quad [1, e^2]$$

a. *(2 marks)* Find the average value of f on the given interval.

b. *(1 mark)* Find c such that $f_{ave} = f(c)$.

c. *(2 marks)* Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.

Question 7. *(5 marks)* Estimate the area under the graph of $f(x) = \tan x$ from $x = -\pi/3$ to $x = \pi/6$ using three rectangles and using the left endpoints. Sketch the curve and the approximating rectangles.

Question 8. *(5 marks)* Prove: If a and b are positive numbers then

$$\int_0^1 x^a(1-x)^b\,dx = \int_0^1 x^b(1-x)^a\,dx$$

Question 9. *(5 marks)* Find a function f such that

$$f'(x) = \frac{1}{\sqrt{1-x^2}}$$

and the line $y = x$ is tangent to the graph f.

Bonus Question. *(3 marks)*
The error function

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

is used in probability, statistics, and engineering.

a. *(1 mark)* Show that

$$\int_a^b e^{-t^2}\, dt = \frac{\sqrt{\pi}}{2}[\operatorname{erf}(b) - \operatorname{erf}(a)].$$

b. *(2 marks)* Show that the function

$$y = e^{x^2}\operatorname{erf}(x)$$

satisfies the differential equation

$$y' = 2xy + \frac{2}{\sqrt{\pi}}$$