Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S09: Winter 2013

Name: _____
Student ID: _____

Test 3

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1995}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$$

Question 2. *(5 marks)* Determine whether the series is convergent or divergent. If it is convergent find its sum.

$$\sum_{n=2}^{\infty} \left[\ln(2n^2+1) - 2\ln(n)\right]$$

Question 3. *(5 marks)* Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=3}^{\infty} \left[\left(\cos\left(2\pi - \frac{1}{n} \right) \right)^{n^2} \right]^n$$

Question 4. *(5 marks)* Set up the integral to find the volume of the solid obtained from the region bounded by the graphs of $x = -y^2 - y^3$, $x = 0$ rotated about the line $y = 1$. Sketch the region, draw a representative rectangle, label all parts of the graph and give the integral.

Question 5. *(5 marks)*

a. *(1 mark)* Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$\left\{\cos 1, \frac{\cos 2}{2}, \frac{\cos 3}{3}, \frac{\cos 4}{4}, \frac{\cos 5}{5}, \ldots\right\}$$

b. *(1 mark)* Is a_n is monotonic?

c. *(1 mark)* Show that a_n is bounded.

d. *(2 marks)* Determine the limit of a_n as $n \to \infty$.

Question 6. *(5 marks)* Determine whether the series is convergent or divergent. If it is convergent find its sum.

$$\sum_{n=2}^{\infty} \frac{2^n - 3^{n-1}}{4^{n+1}}$$

Question 7. *(5 marks)* Determine whether the series is convergent or divergent.

$$\sum_{n=2}^{\infty} \frac{\pi n - \arctan n}{\sqrt{n^5 + n^2 + 1}}$$

Question 8. *(5 marks)* Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=3}^{\infty}(-1)^n\sqrt{\frac{n^4+n}{n^5-n-3}}$$

Question 9. *(5 marks)* Determine whether the series is convergent or divergent. If it is convergent find its sum.

$$\sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{3^n (n!)^2}$$

Bonus Question. *(3 marks)*

a. *(1 mark)* State the $K(\varepsilon)$ definition of the limit of a sequence.

b. *(3 marks)* Show that if $\lim_{n\to\infty} a_{2n} = L$ and $\lim_{n\to\infty} a_{2n+1} = L$, then $\{a_n\}$ is convergent and $\lim_{n\to\infty} a_n = L$

c. *(2 marks)* If $a_1 = 1$ and

$$a_{n+1} = 1 + \frac{1}{1+a_n}$$

converges the find the limit of the sequence.