Name: _____

Student ID: _____

Test 3

This test is graded out of 38 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Compute the indefinite integral*(1 mark each)*:

a.

$$\int \sec x \, dx$$

b.

$$\int \csc x \, dx$$

c.

$$\int \tan x \, dx$$

d.

$$\int \cot x \, dx$$

e.

$$\int e^x \, dx$$

f.

$$\int \frac{1}{x} \, dx$$

g.

$$\int \cos x \, dx$$

Question 2. Compute the indefinite integral.

a. *(2 marks)*

$$\int \frac{x+2}{x^2} \, dx$$

b. *(3 marks)*

$$\int \frac{1-\sin x}{1+\cos x} \, dx$$

c. *(3 marks)*

$$\int \frac{e^{\arcsin 2x}}{\sqrt{1-4x^2}} \, dx$$

Question 3. Compute the definite integral:

a. *(4 marks)*

$$\int_0^{\pi/12} \frac{\sec^2 3x}{4 + \tan 3x} \, dx$$

b. *(4 marks)*

$$\int_0^{1/2} \frac{\ln(2x+3)}{2x+3} \, dx$$

Question 4. *(5 marks)* Sketch and find the area of the region bounded by the graphs of $y = 4 - x^2$, $y = 4x - x^2$, $x = 0$ and $x = 2$.

Question 5. *(5 marks)* Using the disk method, set up the integral to find the volume of the solid obtained by rotating the region bounded by the graphs of the functions $y = x^2 + 1$, $x = 0$, $x = 3$, $y = 0$, about the x-axis. Sketch the region, draw a representative rectangle, write a representative element and the integral.

Question 6. *(5 marks)* Using the shell method, set up the integral to find the volume of the solid obtained by rotating the region bounded by the graphs of the functions $y = \sqrt{4 - x^2}$, in the first quadrant about the y-axis. Sketch the region, draw a representative rectangle, write a representative element and the integral.

Bonus. *(5 marks)* Evaluate the definite integral geometrically:

$$\int_1^5 1 + 2x + 3\sqrt{16 - (x-5)^2}\, dx$$