Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S10: Winter 2016

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Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §8.1 #32 Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{(-3)^n}{n!} = (-1)^n \frac{3^n}{n!} = (-1)^n \frac{3\cdot 3}{1\cdot 2}\left[\frac{3\cdot 3\cdot 3\cdots 3}{3\cdot 4\cdot 5\cdots (n-1)}\right]\frac{3}{n} \le \frac{27}{2n} = b_n$$

< 1

similarly $a_n = (-1)^n \dfrac{3\cdot 3}{1\cdot 2}\left[\dfrac{3\cdot 3\cdot 3\cdots 3}{3\cdot 4\cdot 5\cdots (n-1)}\right]\dfrac{3}{n} \ge \dfrac{-27}{2n}$ c_n

< 1

and $\lim_{n\to\infty} b_n = 0 = \lim_{n\to\infty} c_n$

$\therefore$ by the squeeze thm. $a_n \to 0$ as $n \to \infty$

Question 2. *(5 marks)* §8.2 #25 Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum. If it is convergent, find its sum.

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \sum_{n=2}^{\infty}\left[\frac{A}{n-1} + \frac{B}{n+1}\right] = \sum_{n=2}^{\infty}\left[\frac{1}{n-1} - \frac{1}{n+1}\right]$$

$$\frac{2}{(n-1)(n+1)} = \frac{A}{n-1} + \frac{B}{n+1}$$

$$2 = A(n+1) + B(n-1)$$

Let $n=1$ then $A=1$
$n=-1$ then $A=-1$

$$s_n = a_2 + a_3 + a_4 + a_5 + a_6 + \dots + a_{n-4} + a_{n-3} + a_{n-2} + a_{n-1} + a_n$$

$$= \left[\frac{1}{2-1} - \cancel{\frac{1}{2+1}}\right] + \left[\frac{1}{3-1} - \cancel{\frac{1}{3+1}}\right] + \left[\cancel{\frac{1}{4-1}} - \cancel{\frac{1}{4+1}}\right] + \left[\cancel{\frac{1}{5-1}} - \frac{1}{5+1}\right] + \left[\cancel{\frac{1}{6-1}} - \frac{1}{6+1}\right] + \dots +$$

$$\left[\frac{1}{n-5} - \cancel{\frac{1}{n-3}}\right] + \left[\frac{1}{n-4} - \cancel{\frac{1}{n-2}}\right] + \left[\cancel{\frac{1}{n-3}} - \cancel{\frac{1}{n-1}}\right] + \left[\cancel{\frac{1}{n-2}} - \frac{1}{n}\right] + \left[\cancel{\frac{1}{n-1}} - \frac{1}{n+1}\right]$$

$$= 1 + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}$$

$$s = \lim_{n\to\infty} s_n = \lim_{n\to\infty}\left[1 + \frac{1}{2} - \cancel{\frac{1}{n}}^{\,0} - \cancel{\frac{1}{n+1}}^{\,0}\right] = 3/2$$