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Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §8.3 #28 Determine whether the series is convergent or divergent.

$$\sum_{n=0}^{\infty} \frac{1+\sin n}{10^n} = \sum_{n=0}^{\infty} a_n$$

$0 \le a_n = \frac{1+\sin n}{10^n} \le \frac{1+1}{10^n} = 2\left(\frac{1}{10}\right)^n = b_n$

$\sum_{n=0}^{\infty} b_n$ is a geometric series where $|r| < 1$ $\therefore$ converges.

$\therefore \sum_{n=0}^{\infty} a_n$ converges by comparison test.

Question 2. *(5 marks)* §8.4 #38 Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\frac{2}{5} + \frac{2\cdot 6}{5\cdot 8} + \frac{2\cdot 6\cdot 10}{5\cdot 8\cdot 11} + \frac{2\cdot 6\cdot 10\cdot 14}{5\cdot 8\cdot 11\cdot 14} + \cdots + \frac{2\cdot 6\cdot 10\cdot 14\cdots(4n-2)}{5\cdot 8\cdot 11\cdot 14\cdots(3n+2)} + \cdots = \sum_{n=1}^{\infty} \frac{2\cdot 6\cdot 10\cdot 14\cdots(4n-2)}{5\cdot 8\cdot 11\cdot 14\cdots(3n+2)}$$

Lets apply the ratio test

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{\frac{2\cdot 6\cdot 10\cdot 14\cdots(4n-2)(4(n+1)-2)}{5\cdot 8\cdot 11\cdot 14\cdots(3n+2)(3(n+1)+2)}}{\frac{2\cdot 6\cdot 10\cdot 14\cdots(4n-2)}{5\cdot 8\cdot 11\cdot 14\cdots(3n+2)}} \right|$$

$$= \lim_{n\to\infty} \frac{2\cdot 6\cdot 10\cdot 14\cdots(4n-2)(4n+2)}{5\cdot 8\cdot 11\cdot 14\cdots(3n+2)(3n+5)} \cdot \frac{5\cdot 8\cdot 11\cdot 14\cdots(3n+2)}{2\cdot 6\cdot 10\cdot 14\cdots(4n-2)}$$

$$= \lim_{n\to\infty} \frac{4n+2}{3n+5} = \frac{4}{3} > 1 \quad \therefore \text{diverges by ratio test.}$$