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Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §8.5 #17 Find the radius and interval of convergence for the power series.

$\sum_{n=1}^{\infty} \frac{n}{b^n}(x-a)^n$ where $b > 0$

$\lim_{n\to\infty} \left| \frac{a_{n+1}(x)}{a_n(x)} \right|$

$= \lim_{n\to\infty} \left| \frac{\frac{n+1}{b^{n+1}}(x-a)^{n+1}}{\frac{n}{b^n}(x-a)^n} \right|$

$= \lim_{n\to\infty} \left| \frac{(n+1)(x-a)^{n+1} b^n}{b^{n+1} n (x-a)^n} \right|$

$= \lim_{n\to\infty} \left| \frac{(n+1)(x-a)}{n \; b} \right|$

$= |x-a| \lim_{n\to\infty} \frac{(n+1)}{bn} = \frac{1}{b}|x-a| < 1$

$\frac{1}{b}|x-a| < 1$

$|x-a| < b = R$ $\therefore$ $R = b$.

$-b < x-a < b$

$a-b < x < a+b$

Lets check convergence at the endpoints.

$x = a-b$

$\sum_{n=1}^{\infty} \frac{n}{b^n}(a-b-a)^n = \sum_{n=1}^{\infty} \frac{n}{b^n}(-b)^n = \sum_{n=1}^{\infty} (-1)^n n$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} (-1)^n n$ div. Hence series div. by n^{th} term div. test.

$x = a+b$

$\sum_{n=1}^{\infty} \frac{n}{b^n}(a+b-a)^n = \sum_{n=1}^{\infty} \frac{n}{b^n} b^n = \sum_{n=1}^{\infty} n$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} n$ div. Hence series div. by n^{th} term div. test.

$\therefore$ $(a-b, a+b)$ is the interval of conv.

Question 2. *(5 marks)* §8.7 #15 Find the Taylor series for $f(x) = e^{2x}$ centered at $a = 3$. [Assume that f has a power series expansion. Do not show that $R_n(x) \to 0$.]

$f(x) = e^{2x}$

$f'(x) = 2e^{2x}$

$f''(x) = 2 \cdot 2e^{2x}$

$f'''(x) = 2 \cdot 2 \cdot 2 e^{2x}$

$\vdots$

$f^{(n)}(x) = 2^n e^{2x}$

So $f^{(n)}(3) = 2^n e^{2 \cdot 3}$

$= 2^n e^6$

$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

$= \sum_{n=0}^{\infty} \frac{2^n e^6}{n!}(x-3)^n$