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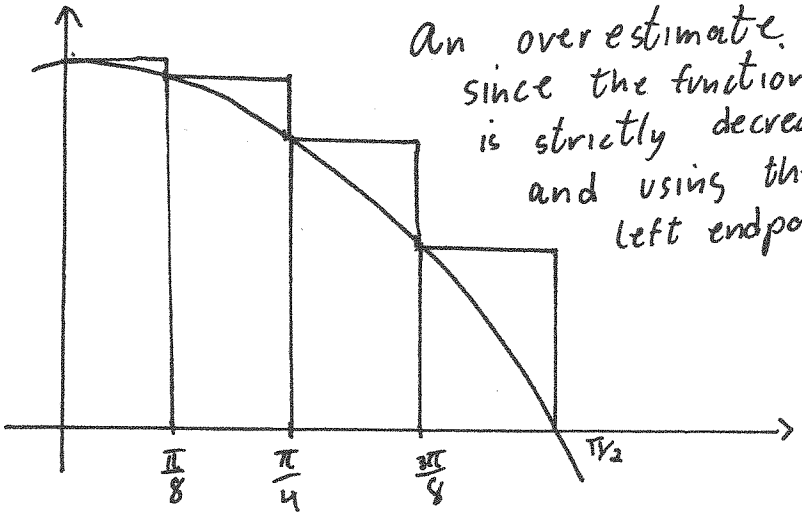
Question 1. *(6 marks)* §5.1 #3b Estimate the area under the graph of $f(x) = \cos x$ from $x = 0$ to $x = \frac{\pi}{2}$ using four approximating rectangles and left endpoints. Sketch the graph and the rectangles. Is your estimate and underestimate or an overestimate?



An overestimate since the function is strictly decreasing and using the left endpoint.

$$\Delta x = \frac{b-a}{n} = \frac{\frac{\pi}{2}-0}{4} = \frac{\pi}{8}$$

$$x_i = a + i\Delta x = \frac{i\pi}{8}$$

$x_0 = 0$

$x_1 = \frac{\pi}{8}$

$x_2 = \frac{\pi}{4}$

$x_3 = \frac{3\pi}{8}$

$x_4 = \frac{\pi}{2}$

$$\text{Area} \approx \sum_{i=1}^{4} f(x_{i-1})\Delta x$$

$$= \frac{\pi}{8}\left[f(x_0) + f(x_1) + f(x_2) + f(x_3)\right]$$

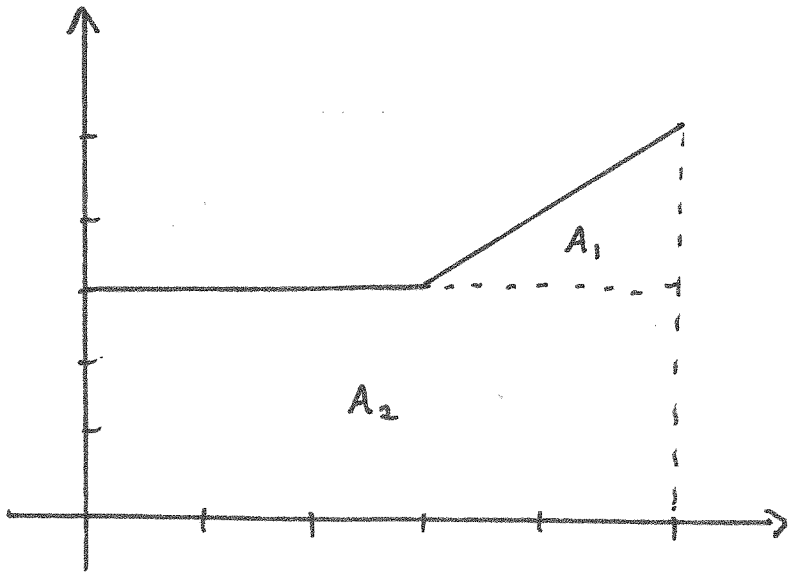
$$= \frac{\pi}{8}\left[f(0) + f(\tfrac{\pi}{8}) + f(\tfrac{\pi}{4}) + f(\tfrac{3\pi}{8})\right]$$

$$= \frac{\pi}{8}\left[\cos 0 + \cos(\tfrac{\pi}{8}) + \cos(\tfrac{\pi}{4}) + \cos(\tfrac{3\pi}{8})\right]$$

$$= \frac{\pi}{8}\left[1 + \cos(\tfrac{\pi}{8}) + \frac{1}{\sqrt{2}} + \cos(\tfrac{3\pi}{8})\right]$$

Question 2. *(5 marks)* §5.2 #42 Find $\int_0^5 f(x)\,dx$ if

$$f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x & \text{for } x \geq 3 \end{cases}$$



$$\int_0^5 f(x)\,dx = A_1 + A_2$$

$$= \frac{1}{2}(2)(2) + 5(3)$$

$$= 17$$