Dawson College: Calculus II (SCIENCE): 201-NYB-05-S10: Winter 2016

Name: Y. Lamontagne

Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §5.5 #68 If f is continuous on $\mathbb{R}$, prove that

$$\int_a^b f(x+c)\,dx = \int_{a+c}^{b+c} f(x)\,dx$$

For the case where $f(x) \geq 0$, draw a diagram to interpret this equation geometrically as an equality of areas.

$$\int_a^b f(x+c)\,dx$$

$u = x+c$

$du = dx$

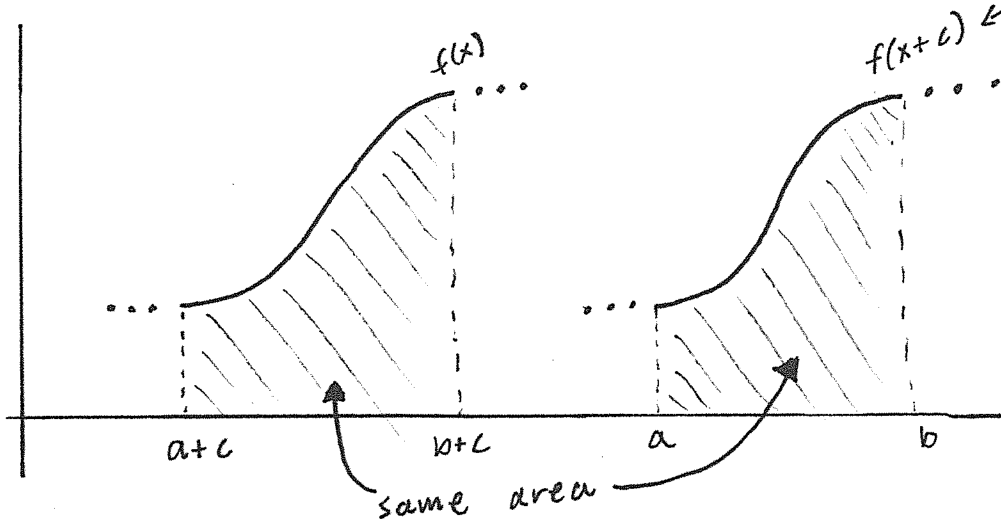
$u(b) = b+c$

$u(a) = a+c$

$$= \int_{a+c}^{b+c} f(u)\,du$$

suppose $a, b > 0$ and $c < 0$

$f(x+c) \leftarrow$ shift of $f(x)$ to the right by $-c$.



Question 2. *(5 marks)* §6.1 #32a Prove the reduction formula

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\,\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$$

$I = \int \cos^n x\,dx$

$I = \int \cos^{n-1} x \cos x\,dx$

$u = \cos^{n-1} x$ $\quad du = (n-1)\cos^{n-2} x\,(-\sin x)\,dx$

$v = \sin x$ $\quad dv = \sin x\,dx$

$I = uv - \int v\,du$

$I = \cos^{n-1}x \sin x - \int (n-1)\cos^{n-2}x\,(-\sin x)\sin x\,dx$

$I = \cos^{n-1}x \sin x + \int (n-1)\cos^{n-2}x \sin^2 x\,dx$

$I = \cos^{n-1}x \sin x + \int (n-1)\cos^{n-2}x\,(1-\cos^2 x)\,dx$

$I = \cos^{n-1}x \sin x + (n-1)\int \cos^{n-2}x - \cos^n x\,dx$

$I = \cos^{n-1}x \sin x + (n-1)\int \cos^{n-2}x\,dx - (n-1)\underbrace{\int \cos^n x\,dx}_{I}$

$I + (n-1)I = \cos^{n-1}x \sin x + (n-1)\int \cos^{n-2}x\,dx$

$nI = \cos^{n-1}x \sin x + (n-1)\int \cos^{n-2}x\,dx$

$I = \frac{1}{n}\cos^{n-1}x \sin x + \frac{(n-1)}{n}\int \cos^{n-2}x\,dx$