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Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §3.7 #38 Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

$y = \lim_{x\to\infty}(e^x+x)^{1/x}$ 	I.f. ∞^0

$\ln y = \ln \lim_{x\to\infty} (e^x+x)^{1/x}$

$\ln y = \lim_{x\to\infty} \ln (e^x+x)^{1/x}$

$\ln y = \lim_{x\to\infty} \frac{1}{x} \ln (e^x+x)$

$\ln y = \lim_{x\to\infty} \frac{\ln(e^x+x)}{x}$ 	I.f. $\frac{\infty}{\infty}$

$\ln y \overset{\hat{H}}{=} \lim_{x\to\infty} \frac{\frac{1}{e^x+x}(e^x+1)}{1}$

$\ln y = \lim_{x\to\infty} \frac{e^x+1}{e^x+x}$ 	I.f. $\frac{\infty}{\infty}$

$\ln y \overset{\hat{H}}{=} \lim_{x\to\infty} \frac{e^x}{e^x+1}$ 	I.f. $\frac{\infty}{\infty}$

$\ln y \overset{\hat{H}}{=} \lim_{x\to\infty} \frac{e^x}{e^x}$

$\ln y = 1$

$e^{\ln y} = e^1$

$y = e$

$\therefore \lim_{x\to\infty} (e^x+x)^{1/x} = e$

Question 2. *(5 marks)* §6.6 #32 Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx = \lim_{a\to 0^+} \int_a^1 \frac{\ln x}{\sqrt{x}}\,dx = \lim_{a\to 0^+}\left[[uv]_a^1 - \int_a^1 v\,du \right]$

$u = \ln x$ 	$du = \frac{1}{x}dx$

$v = 2\sqrt{x}$ 	$dv = \frac{1}{\sqrt{x}}dx$

$= \lim_{a\to 0^+}\left[[2\sqrt{x}\ln x]_a^1 - \int_a^1 \frac{2\sqrt{x}}{x}dx \right]$

$= \lim_{a\to 0^+}\left[\underbrace{2\sqrt{1}\ln 1}_{0} - 2\sqrt{a}\ln a - \int_a^1 \frac{2}{\sqrt{x}}dx \right]$ 	I.f $0\cdot(-\infty)$

$= \lim_{a\to 0^+}\left[\frac{2\ln a}{1/\sqrt{a}} - [4\sqrt{x}]_a^1 \right]$

$\overset{\hat{H}}{=} \lim_{a\to 0^+}\left[\frac{2/a}{\frac{-1}{2a^{3/2}}} - [4\sqrt{1} - 4\cancel{\sqrt{a}}^{\,0}] \right]$

$= \lim_{a\to 0^+}\left[\frac{-4\cancel{a^{3/2}}^{\,0}}{a} - 4 \right] = -4$