Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S10: Winter 2016

Name: ____________________

Test 1

This test is graded out of 46 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

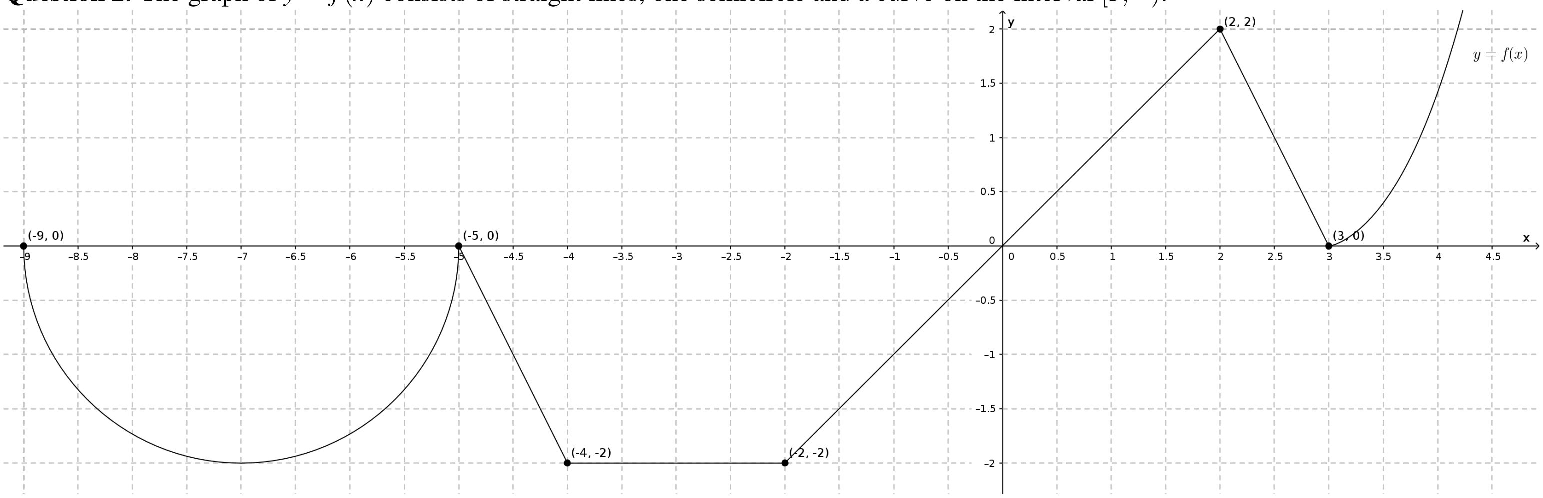
Formulae:

$\sum_{i=1}^{n} c = cn$ where c is a constant $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(5 marks)* Evaluate the definite integral of $f(x) = x^3 + 1$ on $[-1, 2]$ using the definition of the definite integral.

Question 2. The graph of $y = f(x)$ consists of straight lines, one semicircle and a curve on the interval $[3, \infty)$.



a. *(5 marks)* Find an approximation of the area under $f(x)$ on the interval $[0, 2]$, using the right endpoint as sample points and 4 approximating rectangles. Draw the approximating rectangles. Is the approximation an overestimate or underestimate? Justify.

b. *(5 marks)* Evaluate $\int_{-9}^{3} f(x)\, dx$

c. *(5 marks)* If $\int_{-9}^{4} -3f(x) + 2x + 1\, dx = 6\pi - \frac{83}{2}$ then determine $\int_{3}^{4} f(x)\, dx$.

Question 3. *(5 marks)* Evaluate the definite integral:

$$\int_{-\pi/6}^{\pi/4} |\tan(x)|\, dx$$

Question 4. *(5 marks)* Find a function f and a number a such that

$$6 + \int_{a}^{x} \frac{f(t)}{t^2}\, dt = 2\sqrt{x}$$

for all $x > 0$.

Question 5. Given

$$f(x) = \frac{1}{1+x^2}, \quad [0, \sqrt{3}]$$

a. *(2 marks)* Find the average value of f on the given interval.

b. *(2 marks)* Find c such that $f_{ave} = f(c)$.

c. *(2 marks)* Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.

Question 6.

a. *(4 marks)* Prove: If f is an integrable function on $[a,b]$, then

$$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$$

b. *(1 mark)* Prove: If f is an integrable function on $[a,b]$, then

$$\int_a^a f(x)\,dx = 0$$

Question 7. *(5 marks)* Evaluate the indefinite integral

$$\int \frac{(1+\sec(x))^2}{\sec(x)}\,dx$$

Bonus Question. *(3 marks)*
Given the *error function*

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt.$$

Show that the solution to the IVP

$$y' = 2xy + \frac{2}{\sqrt{\pi}}, \quad y(0) = 0$$

is the function

$$y = e^{x^2} erf(x).$$