Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S10: Winter 2016

Name: ________________________

Test 3

This test is graded out of 41 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. For each of the following parts, set up an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis using the specified method. Sketch the region, draw a representative rectangle, write a representative element and label the sketch completely.

$x^2 - y^2 = a^2$, $x = 2a$; about the $x = 4a$ where $a > 0$.

a. *(5 marks)* Using the shell method.

b. *(5 marks)* Using the washer method.

Question 2. Determine whether the statement is true or false. If it is true, explain why. If it is false give an example that disproves the statement.

a. *(2 marks)* If $\{a_n\}$ and $\{b_n\}$ are divergent the $\{a_n + b_n\}$ is divergent.

b. *(2 marks)* If $\{a_n\}$ and $\{b_n\}$ are divergent the $\{a_n b_n\}$ is divergent.

c. *(2 marks)* If $\{a_n\}$ is monotonically decreasing and $a_n > 0$ for all n, then $\{a_n\}$ is convergent.

Question 3. *(5 marks)* Determine if each of the following series converges or diverges. If it converges, find its sum.

$$\sum_{n=1}^{\infty} \ln\left(\frac{2n^2+n+1}{n^2+2n+1}\right)$$

Question 4. *(5 marks)* Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$$

Question 5. *(5 marks)* Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=2016}^{\infty} (-1)^n \frac{\ln(\ln n) + 1}{n \ln n}$$

Question 6. *(5 marks)* Find the radius and interval of convergence for the power series

$$\sum_{n=10!}^{\infty} \frac{(2x-1)^n}{3^n\sqrt{5n+1}}$$

Question 7. *(5 marks)* Find the Maclaurin series for $f(x) = \ln(2+3x)$. [Assume that f has a power series expansion. Do not show that $R_n(x) \to 0$.]

Bonus Question. *(3 marks)*
Using the $K(\varepsilon)$ definition of the limit to prove that if $a_{2n} \to L$ and $a_{2n+1} \to L$ as $n \to \infty$ then $a_n \to L$ as $n \to \infty$.