Determine the convergence of each series. Justify any test that you use.

1. $\displaystyle\sum_{n=5}^{\infty} \frac{3}{n^2-1}$
2. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n+2^n+3^n}{4^n}$
3. $\displaystyle\sum_{n=0}^{\infty} \frac{5^n}{1+2^n+3^n+4^n}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{5+3\cos n}{n}$
5. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt[4]{n^3+1}}$
6. $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt[3]{n^2+2}}{\sqrt{n^3-3}}$
7. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$
8. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^4}$
9. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{e^n}$
10. $\displaystyle\sum_{n=0}^{\infty} n^4 e^{-n}$
11. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n 3^{2n+1}}{n^n}$
12. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}$
13. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{n}$
14. $\displaystyle\sum_{n=1}^{\infty} \frac{\pi^n}{3^n+\sin n-1}$
15. $\displaystyle\sum_{n=0}^{\infty} \frac{n+2^n}{(n+1)2^n}$
16. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(2)(5)\cdots(3n-1)}{3^{n-2}(2)(4)\cdots(2n)}$
17. $\displaystyle\sum_{n=4}^{\infty} \left(\frac{n}{1-2n}\right)^{4n}$
18. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \sqrt{\frac{n}{n^3+1}}$
19. $\displaystyle\sum_{n=2}^{\infty} \frac{n^2+1}{n\ln n}$
20. $\displaystyle\sum_{n=3}^{\infty} \frac{n!\ln n}{(n+1)(2n-1)!}$
21. $\displaystyle\sum_{n=1}^{\infty} \left(2^{1/\sqrt{n}} - 2^{1/\sqrt{n+1}}\right)$
22. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+2}-\sqrt{n-2}}{n}$
23. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n^{100}+1)}{n\ln n}$
24. $\displaystyle\sum_{n=10}^{\infty} \frac{1}{n\ln n\sqrt{(\ln n)^2-1}}$
25. $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{\sqrt{n^7+n}}$
26. $\displaystyle\sum_{n=3}^{\infty} \frac{(n!)^n}{n^{2n}}$
27. $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n^2}}{(n!)^n}$
28. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{e^{n!}}$
29. $\displaystyle\sum_{n=0}^{\infty} \frac{\cos n}{1.01^n}$
30. $\displaystyle\sum_{n=1}^{\infty} n\sin\frac{1}{n}$
31. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n}$
32. $\displaystyle\sum_{n=1}^{\infty} \tan\frac{1}{\sqrt{n^3}}$
33. $\displaystyle\sum_{n=2}^{\infty} \frac{(-3)^{4n+1}(n!)^4}{2^{\ln n}(4n-1)!}$
34. $\displaystyle\sum_{n=0}^{\infty} (-1)^{n+1} \frac{\sqrt[3]{n}+1}{\sqrt[3]{n+1}}$
35. $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n \ln(\arctan n)}{1+n^2}$
36. $\displaystyle\sum_{n=1}^{\infty} \left(1-\frac{1}{3n}\right)^n$
37. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^{\sqrt{n}}}$
38. $\displaystyle\sum_{n=1}^{\infty} \sqrt[n]{n}$
39. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n}}$
40. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}^{\sqrt{n}}}$
41. $\displaystyle\sum_{n=2}^{\infty} \left(\frac{(n-1)!}{n!-1}\right)^n$
42. $\displaystyle\sum_{n=2}^{\infty} \left(1+\frac{(n-1)!}{n!-(n-1)!}\right)^n$
43. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^{\ln n+1}}$
44. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\left(\sqrt[n]{42}-1\right)$
45. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\left(\sqrt[n]{42}-1\right)^n$
46. $\displaystyle\sum_{n=1}^{\infty} \ln\left(1+\frac{\ln n}{n}\right)$
47. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \ln\left(1+\frac{1}{n}\right)$
48. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln(\ln n)+1}{n\ln n}$
49. $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n\sqrt{n}+\ln n}{n}$
50. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n n^2+2n^2}{2^n}$
51. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$

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1. AC[a]
2. AC[b]
3. D
4. D
5. CC
6. D
7. AC
8. AC
9. AC
10. AC
11. AC
12. AC
13. CC
14. D
15. D
16. AC
17. AC
18. CC
19. D
20. AC
21. AC
22. AC
23. D
24. AC
25. AC
26. D
27. AC
28. AC
29. AC[c]
30. D
31. AC
32. AC
33. AC
34. D
35. AC
36. D
37. AC
38. D
39. D[d]
40. AC
41. AC
42. D[e]
43. AC
44. CC
45. AC
46. D
47. CC
48. CC
49. D[f]
50. AC[g]
51. AC

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[a] Show that the sum is 27/40.
[b] Show that the sum is 34/5.
[c] Be careful here – the terms aren't positive and they aren't alternating.
[d] Can you bound $\sqrt[n]{n}$?
[e] Try simplifying the base.
[f] Are the terms always positive? Also note that if $\sum b_n$ diverges, $\sum(a_n+b_n)$ may converge or diverge as well.
[g] Note that we have *not* shown that the sum of two absolutely convergent series is absolutely convergent.