Test 1 (February 22, 2017)
Statistics for Social Science (201-401-DW)
Instructor: Emilie Richer

NAME: SOLUTIONS

Instructions:

- Show all your work. Some solutions will require more written explanation than others. If you use your calculator to compute the mean and SD you do not have to show your work.
- The test is comprised of 8 questions and marked out of a total of **40 marks.**

[QUESTION 1] (5 marks)
Consider the following sample of 25 data points.

0.41	0.32	0.64	0.21	0.29	0.51	0.00	0.19	0.44	0.55	0.31
0.27	0.52	0.40	0.42	0.30	0.11	0.75	0.31	0.10	0.15	0.00
0.41	0.29	0.21								

a. Give the mean and standard deviation for this sample.

$\bar{x} = 0.3244$ $\quad$ $s = 0.187$

b. Give the formula for the sample standard deviation (you do not have to use it to compute the standard deviation in part a)

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

c. Describe the meaning of standard deviation in your own words.

THE STANDARD DEVIATION IS A Measure OF THE "typical" distance of A DATA POINT from THE SAMple Mean.

[QUESTION 2] (5 marks)

Explain the difference between $\bar{x}$ and μ and the difference between **s** and σ in as much detail as possible.

your ANSWER SHOULD CONTAIN THESE elements:

	POPULATION PARAMETER	SAMPLE STATISTIC
COMPUTED USING	all INDIVIDUALS FROM POPULATION OF STUDY	a sample from WITHIN a population
Symbol Mean	μ	$\bar{x}$
STANDARD DEVIATION	σ	s
Formulas	$\sigma = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{N}}$ where N is pop size	$s = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n-1}}$ where n is SAMPLE size
	usually UNKNOWN	KNOWN

[QUESTION 3] (4 marks)

(a) A distribution has a mean of 125 and a standard deviation of 25. A person has a Z-score of 1.29, what is the person's actual score.

$\mu = 125$
$\sigma = 25$
$Z = 1.29$

$$Z = \frac{X-\mu}{\sigma}$$

$$1.29 = \frac{X-125}{25}$$

$$32.25 = X - 125$$

$$X = 157.25$$

(b) What is the meaning of z-score? Describe in your own words.

THE Z-SCORE MEASURES HOW MANY STANDARD DEVIATIONS A POINT IS FROM THE MEAN.

[QUESTION 4] (6 marks)

Friends Brian and Melanie both completed the Montreal marathon, where Brian competed in the Men, Ages 30-34 Group while Melanie competed in the Women, Ages 25-29 Group. Brian completed the race in 3:05:28 (11,128 seconds), while Melanie completed the race in 3:31:53 (12,713 seconds). Here is some information on the performance of their groups:

- The finishing times of the Men, Ages 30-34 Group has a mean of 12,521 seconds with a standard deviation of 1028 seconds.
- The finishing times of the Women, Ages 25-29 Group has a mean of 13,210 seconds with a standard deviation of 1201 seconds.

Remember: **a better performance corresponds to a faster finish.**

(a) What are the Z-scores for Brian's and Melanie's finishing times? What do these Z-scores tell you?

$Z_{BRIAN} = \frac{|11128 - 12521|}{1028} = -1.35$

$Z_{Mel} = \frac{12713 - 13210}{1201} = -0.41$

BOTH Melanie & Brian did better THAN THE AVERAGE OF THEIR respective groups.

(b) Did Brian or Melanie rank better in their respective groups? Explain your reasoning.

Brian ranked Better in his respective group because he is 1.35 STANDARD DEVIATIONS from THE MEAN (AS opposed TO 0.41 FOR Melanie).

IN THIS example NEGATIVE Z-scores INDICATE A Better performance because IT IS BETTER TO TAKE less TIME IN A RACE.

[QUESTION 5] (8.5 marks)

One-hundred and ninety-six (196) teachers were evaluated by their students, the table below summarizes the results of the evaluation:

		Teacher Evaluation					
		Excellent	Very good	Good	Fair	Poor	Total
Teachers' gender	Male	25	29	25	11	10	100
	Female	19	25	24	16	12	96
	Total	44	54	49	27	22	196

(1.5) (a) If we draw one of the evaluated teachers at random what is the probability that the teacher has an *excellent* evaluation and is *male*?

$P(M \text{ \& } exc) = \boxed{25/196}$

(1.5) (b) If we draw one of the evaluated *male* teachers at random, what is the probability that his evaluation is *good* or better?

$P(\text{Good} \cup \text{very good} \cup \text{Excellent} \mid \text{male}) = \frac{25+29+25}{100} = \boxed{\frac{79}{100}}$

(2.5) (c) If we draw one of the evaluated teachers at random, what is the probability that the teacher is *female* **or** that the teacher is evaluated as *excellent*?

$P(F) + P(Exc) - P(F \cap exc) = \frac{96}{196} + \frac{44}{196} - \frac{19}{196} = \boxed{\frac{121}{196}}$

(1.5) (d) If we draw from among the teachers who are evaluated as poor at random, what is the probability that the teacher is male?

$P(M \mid poor) = \frac{10}{22}$

(2) (e) Using the data in the table, determine whether the variables *male* gender and *good* teacher evaluation are independent. Justify?

Let's check if $P(M) = P(M \mid good)$?

$P(M) = \frac{100}{196} = \frac{25}{49}$ $\quad P(M \mid good) = \frac{25}{49}$

The variables are INDEPENDENT, The probability of drawing a Male at random is unaffected by the FACT THAT we KNOW The TEACHER Evaluation is Good

[QUESTION 6] (3 marks)
Consider the two events A = "attending classes regularly" and P = "passing all classes". Would you guess these events to be disjoint? independent? Justify your answer.

THE EVENTS ARE NOT DISJOINT AS A STUDENT CAN SIMULTANEOUSLY "ATTEND CLASS regularly" & "PASS all OF his/her classes"

THE EVENTS ARE NOT INDEPENDENT because The CHANCE OF "PASSING all classes" is affected By "ATTENDING classes regularly".

[QUESTION 7] (6 marks)
Marbles are drawn from an urn, there are 6 red, 4 blue and 3 orange marbles in the urn.

(a) What is the probability of drawing a blue marble from the urn?

$\frac{4}{13}$

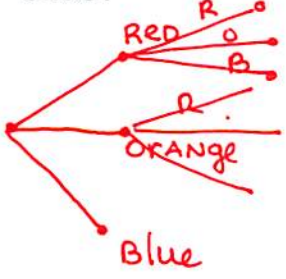
(b) If two marbles are drawn consecutively from the urn *without replacement* and the first marble drawn is blue, what is the probability of drawing a second blue marble?

$\frac{3}{12}$

(c) If two marbles are drawn consecutively from the urn *with replacement* and the first marble drawn is blue, what is the probability that the second marble is blue?

$\frac{4}{13}$

[**BONUS**] (2 marks)
If two marbles are drawn consecutively from the urn *without replacement* what is the probability that the second marble drawn is blue?



WE CAN HAVE Red Blue OR orANge blue or Blue blue

$$P(\text{2ND marble blue}) = P(R \cap B) + P(O \cap B) + P(B \cap B)$$
$$= \left(\frac{6}{13}\right)\left(\frac{4}{12}\right) + \left(\frac{3}{13}\right)\left(\frac{4}{12}\right) + \left(\frac{4}{13}\right)\left(\frac{3}{12}\right) = \frac{4}{13}$$

[QUESTION 8] (2.5 marks)

A card is selected at random from a standard deck of cards. The following events are defined:

A = "the card is a face card"

B = "the card is hearts"

C = "the card is black"

Compute the following probabilities:

(a) $P(A \cap B) = 3/52$

(b) $P(A|C) = 6/26$

(c) $P(A \cap C) = 6/52$

(d) $P(A \cup C) = P(A) + P(C) - P(A \cap C) = \frac{12}{52} + \frac{26}{52} - \frac{6}{52} = \frac{32}{52}$

(e) $P(A|B \cup C) = \frac{9}{39}$