Name: ____________________

Test 1

This test is graded out of 48 marks. No books, notes, watches or cell phones are allowed. You are only permitted to use the Sharp EL-531 calculator. Give the work in full unless otherwise stated, reduce each answer to its simplest exact form. Write and arrange your exercise in a legible and orderly manner. If you need more space for your answer use the back of the page.

Question 1. Given

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 2 & 1 \\ -3 & 2 & 1 \end{bmatrix}.$$

a. *(5 marks)* Find all 3×3 lower triangular matrices B such that $AB = 0$.

b. *(1 mark)* List two such matrices B from part a.

Question 2. Given the matrix $A = \begin{bmatrix} 1 & x \\ 0 & -x \end{bmatrix}$,

a. *(2 marks)* Calculate A^2 and A^3

b. *(2 marks)* Give an expression for A^n, where n is any natural number.

c. *(3 **bonus** marks)* Given the matrix $Q = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, and $|x| < 1$ calculate $\lim_{n \to \infty} (Q + A^n)$

Question 3. *(5 marks)* Express $\begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$ and its inverse as products of elementary matrices.

Question 4. *(4 marks)* Let $A\mathbf{x} = \mathbf{0}$ be a homogeneous system of n linear equations in n unknowns, and let Q be an invertible $n \times n$ matrix. Prove that $A\mathbf{x} = \mathbf{0}$ has only the trivial solution if and only if $(QA)\mathbf{x} = \mathbf{0}$ has only the trivial solution.

Question 5. *(5 marks)* Given

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

solve for X, if possible.

$$E_3\left(E_1 X + E_1 A\right) = E_2 A$$

Question 6. *(3 marks)* If A is a symmetric $n \times n$ matrix and P is any $m \times n$ matrix, show that PAP^T is symmetric.

Question 7. *(1 mark each)* Circle the correct answer. Given A an $n \times n$ matrix and k a non-zero scalar.

a. A is an elementary matrix obtained by interchanging two rows.

i. $\det(A) = -1$
ii. $\det(A) = 0$
iii. $\det(A) = 1$
iv. $\det(A) = k$
v. $\det(A) = n$
vi. $\det(A) = kn$
vii. $\det(A) = k^n$
viii. $\det(A) = n^k$
ix. The determinant of A is not necessarily equal to one of the other options.

b. A is the reduced row echelon form of an invertible matrix.

i. $\det(A) = -1$
ii. $\det(A) = 0$
iii. $\det(A) = 1$
iv. $\det(A) = k$
v. $\det(A) = n$
vi. $\det(A) = kn$
vii. $\det(A) = k^n$
viii. $\det(A) = n^k$
ix. The determinant of A is not necessarily equal to one of the other options.

c. A is a singular matrix.

i. $\det(A) = -1$
ii. $\det(A) = 0$
iii. $\det(A) = 1$
iv. $\det(A) = k$
v. $\det(A) = n$
vi. $\det(A) = kn$
vii. $\det(A) = k^n$
viii. $\det(A) = n^k$
ix. The determinant of A is not necessarily equal to one of the other options.

d. A is an elementary matrix obtained by adding k times one row to an other.

i. $\det(A) = -1$
ii. $\det(A) = 0$
iii. $\det(A) = 1$
iv. $\det(A) = k$
v. $\det(A) = n$
vi. $\det(A) = kn$
vii. $\det(A) = k^n$
viii. $\det(A) = n^k$
ix. The determinant of A is not necessarily equal to one of the other options.

e. A is an elementary matrix obtained by multiplying one row by k.

i. $\det(A) = -1$
ii. $\det(A) = 0$
iii. $\det(A) = 1$
iv. $\det(A) = k$
v. $\det(A) = n$
vi. $\det(A) = kn$
vii. $\det(A) = k^n$
viii. $\det(A) = n^k$
ix. The determinant of A is not necessarily equal to one of the other options.

f. A is the identity matrix multiplied by k.

i. $\det(A) = -1$
ii. $\det(A) = 0$
iii. $\det(A) = 1$
iv. $\det(A) = k$
v. $\det(A) = n$
vi. $\det(A) = kn$
vii. $\det(A) = k^n$
viii. $\det(A) = n^k$
ix. The determinant of A is not necessarily equal to one of the other options.

Question 8. *(5 marks)* Compute the determinant below by only using co-factor expansions.

$$\begin{vmatrix} 1 & 2 & 0 & 3 \\ -3 & 2 & 1 & 0 \\ 5 & -9 & 0 & 2 \\ 8 & -7 & -3 & 1 \end{vmatrix}$$

Question 9. *(5 marks)* Let A and B be 5×5 matrices such that $(A^3B)\text{adj}(2A) = \frac{1}{4}A^T$ and $\det(B) = 7$. Find $\det(A)$.

Question 10. *(5 marks)* Prove the following identity by only using properties of determinants and elementary operations *(do not evaluate the determinants directly)*.

$$(k^2-1)\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} g(1-k) & h+i & i \\ d(1-k^2) & (e+f)(1+k) & f(1+k) \\ a(1-k) & b+c & c \end{vmatrix}$$