Dawson College: Winter 2019: Linear Algebra (SCIENCE): 201-NYC-05-S6: **Quiz 6** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* If $A = \begin{bmatrix} k & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, show that $\det(AA^T) \neq \det(A^TA)$ for every $k \in \mathbb{R}$.

$$AA^T = \begin{bmatrix} K & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} K & 0 \\ 3 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} K^2+10 & 3 \\ 3 & 1 \end{bmatrix} \qquad |AA^T| = K^2+1$$

$$A^TA = \begin{bmatrix} K & 0 \\ 3 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} K & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} K^2 & 3K & K \\ 3K & 10 & 3 \\ K & 3 & 1 \end{bmatrix}$$

$$|A^TA| = K^2 \begin{vmatrix} 10 & 3 \\ 3 & 1 \end{vmatrix} - 3K \begin{vmatrix} 3K & 3 \\ K & 1 \end{vmatrix} + K \begin{vmatrix} 3K & 10 \\ K & 3 \end{vmatrix}$$

$$= K^2 - K^2 = 0$$

So $|AA^T| = |A^TA|$

$K^2+1 = 0$ no $K \in \mathbb{R}$ satisfy the equation.

Question 2. *(5 marks)* Use elementary operations to show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$$

$$\text{LHS} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} \underset{\substack{-aR_1+R_2 \to R_2 \\ -a^2R_1+R_3 \to R_3}}{=} \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & b^2-a^2 & c^2-a^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & (b-a)(b+a) & (c-a)(c+a) \end{vmatrix}$$

$$\underset{-(b+a)R_2+R_3 \to R_3}{=} \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & 0 & (c-a)(c+a)-(c-a)(b+a) \end{vmatrix}$$

$$= 1(b-a)\left[(c-a)(c+a)-(c-a)(b+a)\right]$$

$$= (b-a)(c-a)\left[c+a-(b+a)\right]$$

$$= (b-a)(c-a)(c-b)$$

Question 3. *(5 marks)* Find the determinant of the matrix A.

$$A = \begin{bmatrix} 1 & \det(A) & 1 & a \\ -2 & 3 & 1 & b \\ 2 & 5 & 0 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\det(A) = a_{41}C_{41} + a_{42}C_{42} + a_{43}C_{43} + a_{44}C_{44}$$

$$\det(A) = 0 + 0 + 0 + 1(-1)^{4+4} \begin{vmatrix} 1 & \det(A) & 1 \\ -2 & 3 & 1 \\ 2 & 5 & 0 \end{vmatrix}$$

$$\det(A) = a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33}$$

$$\det(A) = \begin{vmatrix} -2 & 3 \\ 2 & 5 \end{vmatrix} - \begin{vmatrix} 1 & \det(A) \\ 2 & 5 \end{vmatrix}$$

$$\det(A) = -10-6-[5-2\det(A)]$$

$$-\det(A) = -21$$

$$\det(A) = 21$$