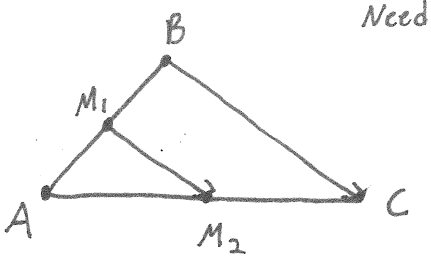
Dawson College: Winter 2019: Linear Algebra (SCIENCE): 201-NYC-05-S6: **Quiz 8** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Using vectors prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and half as long.



Need to show: $\frac{1}{2}\vec{BC} = \vec{M_1M_2}$

$RHS = \vec{M_1M_2}$

$= \vec{M_1B} + \vec{BC} + \vec{CM_2}$

$= \frac{1}{2}\vec{AB} + \vec{BC} + \frac{1}{2}\vec{CA}$

$= \frac{1}{2}[\vec{AB} + \vec{CA}] + \vec{BC}$

$= \frac{1}{2}[\vec{CA} + \vec{AB}] + \vec{BC}$

$= \frac{1}{2}[\vec{CB}] + \vec{BC} = \frac{1}{2}[-\vec{BC}] + \vec{BC} = \frac{1}{2}\vec{BC} = LHS$

Question 3. *(5 marks)* Prove the parallelogram law for the norm:

$$||\vec{a}+\vec{b}||^2 + ||\vec{a}-\vec{b}||^2 = 2||\vec{a}||^2 + 2||\vec{b}||^2$$

for all vectors in $\mathbb{R}^n$.

$LHS = ||\vec{a}+\vec{b}||^2 + ||\vec{a}-\vec{b}||^2$

$= (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) + (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$

$= \vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b} + \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b}$

$= ||\vec{a}||^2 + ||\vec{b}||^2 + ||\vec{a}||^2 + ||\vec{b}||^2$

$= 2||\vec{a}||^2 + 2||\vec{b}||^2$

$= RHS$

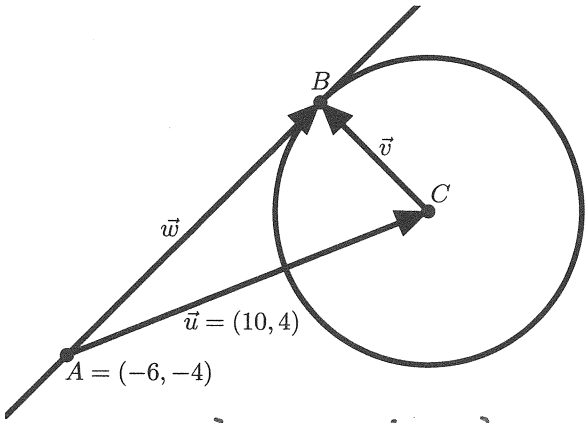
Question 3. Given the line $y = x + 2$ which is a tangent of the circle with centre C. The vector $\vec{v}$ has initial point C and terminal point B which lies both on the circle and the tangent.

note: A tangent to a circle is perpendicular to the radius at the point at which the tangent and circle intersect.

a. *(1 mark)* Find the centre C of the circle.

b. *(3 marks)* Find the point B.

d. *(1 mark)* Find the equation of the circle.



a) $\vec{AC} = \vec{u}$

$\vec{u} = \vec{AC}$

$(10,4) = C - A$

$(10,4) = C - (-6,-4)$

$C = (4,0)$

b) $B(x, x+2)$ since it lies on the line.

$\vec{AB} \perp \vec{CB}$ by note.

So $\vec{AB}\cdot\vec{CB} = 0$.

$\vec{AB} = B - A = (x, x+2) - (-6,-4)$

$= (x+6, x+6)$

$\vec{CB} = B - C = (x, x+2) - (4, 0)$

$= (x-4, x+2)$

$0 = \vec{AB}\cdot\vec{CB}$

$0 = (x+6, x+6)\cdot(x-4, x+2)$

$0 = (x+6)(x-4) + (x+6)(x+2)$

$0 = x^2 + 6x - 4x - 24 + x^2 + 2x + 6x + 12$

$0 = 2x^2 + 10x - 12$

$0 = x^2 + 5x - 6$

$0 = (x+6)(x-1)$

$x = 6$ $\quad$ $x = 1$

$x = -6$ makes $\vec{AB} = \vec{0}$

Hence $x = 1 \therefore B(1, 3)$.

c) $(x-h)^2 + (y-k)^2 = r^2$

$(x-4)^2 + y^2 = ||\vec{u}||^2$

$(x-4)^2 + y^2 = ||(-3,3)||^2$

$(x-4)^2 + y^2 = \left(\sqrt{(-3)^2+3^2}\right)^2$

$(x-4)^2 + y^2 = 18$