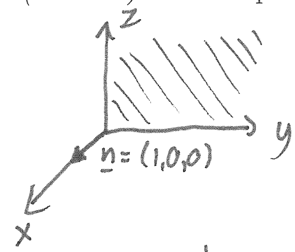
Dawson College: Winter 2019: Linear Algebra (Science): 201-NYC-05-S6: **Quiz 9** name: I. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] Given the points $A(-2,0,-1)$ and $B(-2,1,0)$.

a. *(5 marks)* Find the point C, if possible, on the yz-plane such that the points A, B and C are collinear.



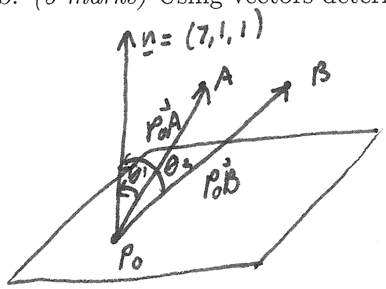
The parametric equation of line that contains A and B is: $\underline{x} = A + t\vec{AB}$ $t \in \mathbb{R}$ $\vec{AB} = B - A = (-2,1,0) - (-2,0,-1) = (0,1,1)$

$(x,y,z) = (-2,0,-1) + t(0,1,1)$
$= (-2, t, -1+t)$

So intersection between the line and plane does not exist since $0 = -2$ can not be satisfied.

$ax+by+cz=d$
$x = d$
sub-in $(0,0,0)$
$x = 0$
$\therefore$ equation of yz-plane $x=0$

b. *(5 marks)* Using vectors determine whether the points A and B are on the same side of the plane $7x+y+z=1$.



Let $x=y=0 \Rightarrow z=1$ $\therefore P_0(0,0,1)$

$\vec{P_0A} = A - P_0 = (-2,0,-1) - (0,0,1) = (-2,0,-2)$

$\vec{P_0B} = B - P_0 = (-2,1,0) - (0,0,1) = (-2,1,-1)$

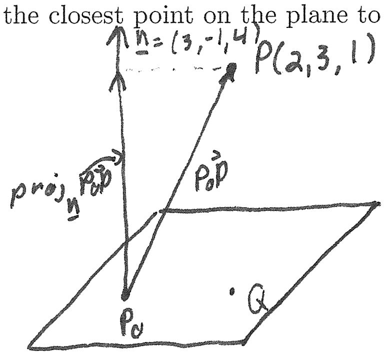
$\cos\theta =$

$\vec{P_0A} \cdot \vec{n} = (-2,0,-2)\cdot(7,1,1) = -16 < 0 \therefore \theta_1$ is obtuse

$\vec{P_0B} \cdot \vec{n} = (-2,1,-1)\cdot(7,1,1) = -15 < 0 \therefore \theta_2$ is obtuse

$\therefore$ A and B are on the same side of the plane.

Question 3. *(5 marks)* Use a projection(s) to find the distance from the point $P(2,3,1)$ to the plane $3x_1 - x_2 + 4x_3 = 5$. And find the closest point on the plane to the given point. Let $x_1 = x_3 = 0 \Rightarrow x_2 = -5$ $P_0(0,-5,0)$



$\vec{P_0P}$
$= P - P_0$
$= (2,3,1) - (0,-5,0)$
$= (2,8,1)$

$\text{proj}_{\underline{n}} \vec{P_0P}$

$= \dfrac{\underline{n}\cdot\vec{P_0P}}{\underline{n}\cdot\underline{n}}\,\underline{n}$

$= \dfrac{(3,-1,4)\cdot(2,8,1)}{(3,-1,4)\cdot(3,-1,4)}(3,-1,4)$

$= \dfrac{2}{26}(3,-1,4)$

$= \dfrac{1}{13}(3,-1,4)$

distance $= \|\text{proj}_{\underline{n}} \vec{P_0P}\|$
$= \|\frac{1}{13}(3,-1,4)\|$

$= \frac{1}{13}\|(3,-1,4)\|$
$= \frac{1}{13}\sqrt{26}$

$\vec{QP} = \text{proj}_{\underline{n}} \vec{P_0P}$

$P - Q =$ "

$Q = P - \text{proj}_{\underline{n}} \vec{P_0P}$

$= (2,3,1) - \frac{1}{13}(3,-1,4)$

$= \left(\frac{23}{13}, \frac{40}{13}, \frac{9}{13}\right)$

[1] From a past Dawson College Final Examination