Dawson College: Winter 2022: Calculus II (SCIENCE): 201-NYB-05-S1: **Test 1, part 1 of 3** name: ____________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show and justify all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Formulae:

$\sum_{i=1}^{n} c = cn$ where c is a constant $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. Calculus III ____________ be extremely fun.

b. Suppose $f(x)$ be a continuous odd function on the interval $[-2, 2]$ then $\int_{-1}^{2} f(x)\, dx$ ____________ be equal to zero.

c. The mean value theorem for integrals states that if $f(x)$ is continuous on $[a, b]$ then there ____________ exists a number c in $[a, b]$ such that $\int_a^b f(x)\, dx = f(c)(b - a)$.

d. $\int_a^b f(x)\, dx$ ____________ be equal to $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$ where $\Delta x = (b - a)/n$ and $x_i = a + i\Delta x$.

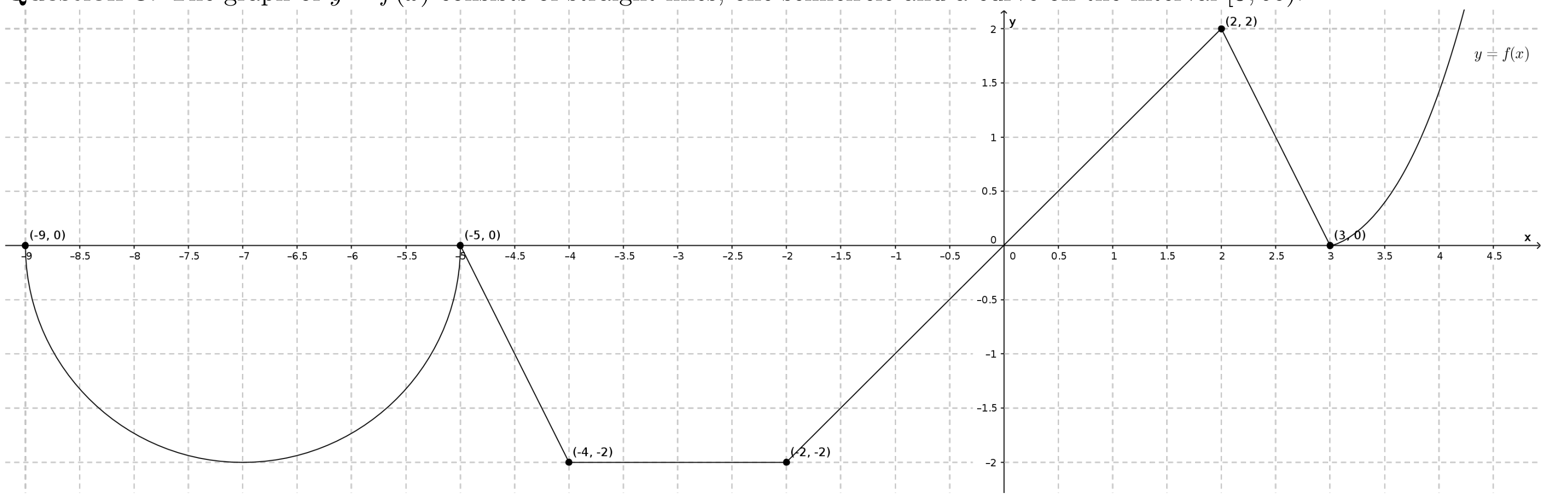
e. $\int_a^b f(x)\, dx$ ____________ be equal to $\int_a^b f(\alpha)\, d\alpha$.

Question 2. *(2 marks)* Suppose $a_m, a_{m+1}, \ldots, a_n$ and k are real numbers, prove $\sum_{i=m}^{n} ka_i = k \sum_{i=m}^{n} a_i$.

Question 3a. *(2 marks)* State the entire Fundamental Theorem of Calculus (FTC).

Question 3b. *(2 marks)* As seen in class the *Net Change Theorem* is an application of a part of the FTC. Explain the Net Change Theorem using an example.

Question 3. The graph of $y = f(x)$ consists of straight lines, one semicircle and a curve on the interval $[3, \infty)$.



a. *(5 marks)* Find an approximation of the definite integral of $f(x)$ on the interval $[-4, 2]$, using the left endpoint as sample points and 4 approximating rectangles. Draw the approximating rectangles. Is the approximation an overestimate or underestimate? Justify.

b. *(2 marks)* Evaluate $\int_{-4}^{2} f(x)\, dx$.

c. *(4 marks)* Evaluate $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n} + f\left(-4 + \frac{6i}{n}\right)\right) \frac{6}{n}$.

Question 4. Given the function

$$f(x) = \frac{1}{2}\int_{e}^{x^2} \frac{\arctan(\ln t)}{t}\, dt.$$

a. *(5 marks)* Evaluate $f(x)$.

b. *(5 marks)* Determine whether $f(x)$ is a solution to the initial value problem (IVP) given below

$$y' = \arctan(2\ln x), \quad y\left(\sqrt{e}\right) = 0.$$

Question 5. *(5 marks)* Find the average of the function $f(x) = |\tan^3(x)\sec^3(x)|$ on $[-\pi/6, \pi/6]$.

Question 6. [1] *(5 mark each)* Evaluate the following integrals:

a.

$$\int_0^{-\ln(2)} e^x \sqrt{2e^x - e^{2x}}\, dx$$

[1]From or modified from a John Abbott final examination

b.

$$\int \frac{x^3 + x^2 + x + 2}{(x+1)(x^2+1)}\, dx$$

Bonus Question. *(5 marks)* Use the following integral

$$\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$$

to show that $\pi < \frac{22}{7}$.