Dawson College: Winter 2022: Linear Algebra (Science): 201-NYC-05-S7: **Test 1, part 1 of 2** name: ____________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. Let A be a square matrix. $A\mathbf{x} = A\mathbf{y}$ only if $\mathbf{x} = \mathbf{y}$, then A ______________ be invertible.

b. A linear system with three equations and two variables ______________ be inconsistent.

c. If A, B are matrices such that $AB = I$ then matrix A ______________ be invertible.

d. If AB is defined, then BA ______________ also defined.

e. If A and B are $n \times n$ matrices such that $AB = B$, then A ______________ be an identity matrix.

f. If A and B are both square matrices such that AB equals BA equals the identity matrix, then B ______________ the inverse matrix of A.

g. If E_1 and E_2 are two elementary matrices, then E_1E_2 ______________ be equal to E_2E_1.

h. For any invertible matrix A, the number of leading ones of the RREF of A ______________ be the same as the number of leading ones of the RREF of A^2.

Question 2.[1] *(5 marks)* Given X, B, and C are $n \times n$ matrices, solve the following equation for X. Assume any necessary matrices to be invertible.

$$(3X^{-1}B)^{-1} = C(X + B)$$

Question 3.[2] *(5 marks)* Find **all** matrices A, if any, such that $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A + A \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 2 & 2 \end{bmatrix}$

[1] From or modified from a John Abbott final examination or WeBWorK

[2] From a Dawson final examination

Question 4. *(5 marks)* Given

$$A = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 2 & -2 & 0 & 3 \\ 3 & 0 & 0 & -2 \\ 5 & -2 & -1 & 0 \end{bmatrix}$$

a. *(5 marks)* Find the reduced row echelon form of the matrix A.

b. *(2 marks)* Suppose that A is the coefficient matrix of a homogeneous linear system. Write the corresponding system of linear equations. Use part a. to find the solution of the system.

c. *(1 mark)* Suppose that A is the coefficient matrix of a homogeneous linear system. Use part c. to find two particular solutions to the system.

d. *(4 marks)* Determine whether or not it is possible to express A as product of elementary matrices times the RREF of A (i.e. $A = F_1 F_2 \dots F_k R$ where the F_i are elementary matrices and R is the RREF of A). If it is give F_1 and F_k. Justify.

Question 5.[1] *(5 marks)* Let $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & k & 6 \\ 0 & 2 & 4k \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 3h \\ 2 \end{bmatrix}$. Find the value(s) of h and k, if possible, for which the equation $A\mathbf{x} = \mathbf{b}$ has:

a. a unique solution,

b. infinitely many solutions,

c. no solution.

Question 6. A square matrix A is *idempotent* if $A^2 = A$.

a. *(1.5 marks)* Find three idempotent matrices.

b. *(1.5 marks)* Prove that if A is idempotent then A^T is idempotent.

Question 7.[3] *(5 marks)* Prove that if A and B are $m \times n$ matrices, then A and B are row equivalent if and only if A and B have the same reduced row echelon form.

Question 8. *(3 marks)* If A is symmetric and skew-symmetric then $A = 0$.

Bonus Question. *(5 marks)* Enumerate all the solution(s) of $A\mathbf{x} = \mathbf{0}$ where

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 2 & 0 & 2 \end{bmatrix}$$

given that the numbers are $\mathbb{Z}_3$ instead of $\mathbb{R}$. Operations on the numbers of $\mathbb{Z}_3$ can be defined by the following Cayley tables:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

×	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

[3] From assigned homework.