Dawson College: Winter 2022: Linear Algebra (SCIENCE): 201-NYC-05-S8: **Test 2, part 1 of 2** name: ______________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(1 mark per blank)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. If A is a product of elementary matrices, then $\det(A)$ ______________ equal zero..

b. Let A be a 3×3 matrix, and let B be a 4×4 matrix. If the leading ones of the RREF of A is equal to those of the RREF of B, then $\det(B)$ ______________ equal zero and $det(A)$ ______________ equal zero.

c. Two lines in $\mathbb{R}^3$ that are both perpendicular to a third line ______________ be parallel.

d. If $\vec{u}$ and $\vec{v}$ are nonzero vectors in $\mathbb{R}^3$, then $(\vec{u} \times \vec{v}) \cdot \vec{u}$ ______________ be equal to 0.

e. Let $\vec{u}$ be parallel to $\vec{x}$, and let $\vec{v}$ be parallel to $\vec{y}$. Then $\vec{u} + \vec{v}$ ______________ be parallel to $\vec{x} + \vec{y}$.

f. The vector $\vec{u} \times (\vec{v} \times \vec{w})$ ______________ be a solution of $\vec{v} \cdot \vec{x} = 0$ and $\vec{w} \cdot \vec{x} = 0$.

Question 2. *(1 mark per blank)* Given A an $n \times n$ matrix and k a non-zero scalar.

a. If A is an elementary matrix obtained by interchanging two rows then $\det(A) =$ ______________.

b. If A is a matrix which is obtained by multiplying each row of the identity by the number of the row then $\det(A) =$ ______________.

c. If A is an elementary matrix obtained by multiplying one row by k then $\det(A) =$ ______________.

Question 3.[2] *(5 marks)* If A and B are invertible matrices of the same size show that

$$\operatorname{adj}(AB) = \operatorname{adj}(B)\operatorname{adj}(A)$$

.

Question 4. *(5 marks)* Find the determinant of the matrix A.

$$A = \begin{bmatrix} 2 + 2\operatorname{trace}(A) & \det(A) & 1 \\ -2 & 3 & 1 \\ 2 & 5 & 0 \end{bmatrix}$$

[1] From or modified from a John Abbott final examination

[2] From a Dawson College final examination

Question 5.[1] *(5 marks)* Given that $\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = 10$ and $A = \begin{bmatrix} 3g+a & 3h+b & 2 & 3i+c \\ d+2a & e+2b & 3 & f+2c \\ a & b & 4 & c \\ 0 & 0 & 5 & 0 \end{bmatrix}$

a. *(5 marks)* Find $\det(A)$.

b. *(3 marks)* Using Cramer's Rule find x_1 and x_3 for $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{bmatrix} 2a & 3a & 4a & 5a \end{bmatrix}^T$

Question 6.[1] *(3 mark)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

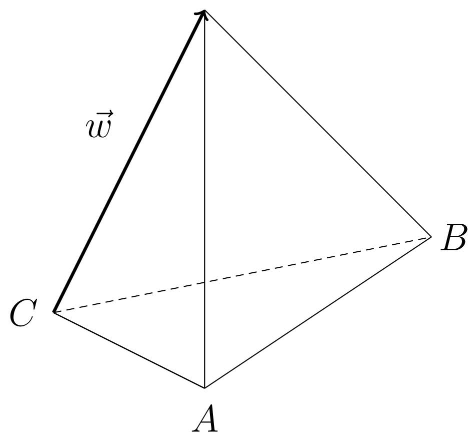
If A is a skew-symmetrix $n \times n$ matrix where n is odd then $\det(A) = 0$.

Question 7.[1] *(3 marks)* Show that if $\text{proj}_{\vec{v}}\,\vec{u} = \text{proj}_{\vec{v}}\,\vec{w}$, then $\vec{u} - \vec{w}$ is orthogonal to $\vec{v}$.

Question 8. *(2 marks)* Sketch $\vec{v} = (2, 3, 5)$ as shown in class, include and label the axes.

Question 9. *(5 marks)* Given the terahedron determined by the points $A(2, -1, -1)$, $B(2, -1, -2)$, $C(0, 8\sqrt{5} - 1, 0)$ and the vector $\vec{w} = (4, 1, 3)$. Note that from the diagram, $\vec{w}$ is not perpendicular to the base.

Find the volume of the tetrahedron. *(Hint: the volume of a tetrahedron is equal to one third of the area of the base times the height.)*



Question 10. Given the plane $x + y + z = 0$ and the line $(x,\ y,\ z) = (1 + t, 2 + 2t, 3 + 3t)$ where $t \in \mathbb{R}$.

a. *(2 marks)* Determine whether the line is perpendicular to the plane, parallel or neither. Justify

b. *(2 marks)* Find the point of intersection between the line and the plane if it exists.

Question 11.[1] Given $\mathcal{L}_1 : \vec{x} = (1, 0, 1) + t(-4, -2, 6)$, where $t \in \mathbb{R}$.

a. *(4 marks)* Find an equation for the line through the origin that intersects $\mathcal{L}_1$ at a right angle.

b. *(2 marks)* Find the distance between the origin and $\mathcal{L}_1$.

c. *(2 marks)* Find the closest point on $\mathcal{L}_1$ to the origin.

c. *(2 marks)* Find the plane that contains $\mathcal{L}_1$ and the origin.

Question 12.[3]

a. *(1 marks)* Find a homogeneous linear system of two equations in three unknowns whose solution space consists of those vectors in $\mathbb{R}^3$ that are orthogonal to $\vec{a} = (-3, 2, -1)$ and $\vec{b} = (0, -2, -2)$.

b. *(2 marks)* What kind of geometric object is the solution space? Justify.

c. *(2 marks)* Find a vector which is parallel to the solution space without solving the system. Using that vector find the solution space.

Bonus Question. *(5 marks)* If $\vec{u}, \vec{v} \in \mathbb{R}^n$ then $|\vec{u} \cdot \vec{v}| \leq ||\vec{u}||||\vec{v}||$. *Hint: Analyse the squared norm of* $||\vec{u}||\vec{v} - ||\vec{v}||\vec{u}$ *and* $||\vec{u}||\vec{v} + ||\vec{v}||\vec{u}$.

[3]From the assigned homework.